



SIXTH FORM INDUCTION WEEK 2nd - 5th JULY 2018

SUMMER TASK - MATHS A LEVEL

To All Prospective A Level Mathematics Students:

You will appreciate that Mathematics is a subject which relies to a significant extent on two elements:

1. Natural flair and aptitude;
2. Consolidation and practice of the topics within each course.

As you move from GCSE to A level, and hopefully beyond, it is absolutely essential that your progress is not unduly hampered by weaknesses in basic algebraic techniques for example.

In this document is a pack of six topics which you are required to work through during the summer break. It represents some of the more challenging GCSE topics, which are termed '**assumed knowledge**' by the A Level examination board. If you have any problems you must seek help immediately at the start of the summer term.

You will be tested on this work in September. Students who fail to reach a minimum standard will be required to do additional work and their potential to continue the course will be reviewed. Parents will be contacted.

LIST OF TOPICS

1. Factorising
2. Formulae
3. Linear and Quadratic Equations
4. Simultaneous Equations
5. Simplifying, including indices
6. The Sine Rule and Cosine Rule

SUBMISSION DATE – THURSDAY 6TH SEPTEMBER 2018

TOPIC 1 : Factorising

Examples

1. $2x^3 - 6x^2 = 2x^2(x - 3)$
2. $9a^2 - 4b^2 = (3a + 2b)(3a - 2b)$
3. $x^2 - 7x + 12 = (x - 4)(x - 3)$
4. $6x^2 + 11x + 4 = (2x + 1)(3x + 4)$

Exercise

Factorise completely:

1. $n^2 - np$
2. $h^2 - 25$
3. $m^2 + 7m + 10$
4. $n^2 - n - 12$
5. $15 - 2b - b^2$
6. $3x^2 - 75$
7. $5h^2 - 8h - 4$
8. $10x^2 + 9x + 2$
9. $ab + 5a - 2b - 10$
10. $(x + 3)(x + 5) + (x + 3)^2$

SUBMISSION DATE – THURSDAY 6TH SEPTEMBER 2018

TOPIC 2 : Formulae

A. Re-arranging formulae

Examples

In each case, re-arrange to find the letter in the bracket.

1.

$$y = \frac{x}{2} + 7 \quad (x)$$

$$2y = x + 14$$

$$x = 2y - 14$$

2.

$$y = 4(x + 3) \quad (x)$$

$$\frac{y}{4} = x + 3$$

$$x = \frac{y}{4} - 3$$

3.

$$a^2 = b^2 - 7ac \quad (b)$$

$$b^2 = a^2 + 7ac$$

$$b = \pm\sqrt{a^2 + 7ac}$$

4.

$$\frac{3}{m} = 6 - 4n \quad (m)$$

$$3 = (6 - 4n)m$$

$$m = \frac{3}{6 - 4n}$$

Exercise

Re-arrange each formula to make the letter in the bracket the subject.

1. $y = \sqrt{3x} \quad (x)$

2. $4y = 2x - 7 \quad (x)$

3. $3(y + 2) = 6 - 3(x + 7) \quad (x)$

4. $ab - cd = 4e \quad (c)$

5. $3a(x + y) = 2b^2 \quad (a)$

6. $\frac{3y}{x} = \frac{x}{4z} \quad (x)$

7. $bx + cy = d^2 \quad (c)$

8. $k(l - m) = l(m - n) \quad (l)$

B. Substitution in formulae

Examples

Find the value of x if $a = 0.7$, $b = -3.5$ and $c = -2.15$. Give answers to 3 sig. figs.

1.

$$x = ab - c^2$$

$$x = 0.7 \times (-3.5) - (-2.15)^2$$

$$x = -7.07$$

2.

$$x = \sqrt{a(b^2 + c)}$$

$$x = \sqrt{0.7((-3.5)^2 - 2.15)}$$

$$x = 2.66$$

Exercise

Use the above values to find x in each case, correct to 3 sig. Figs.:

1. $x = 4bc + a^3$

2. $x = \frac{ac + b^2}{c}$

3. $x = \sqrt{b^2 - \frac{c}{a}}$

4. $7x = 2a - 3b + 4c$

5. $ax + bx = c$

TOPIC 3 : Linear and Quadratic Equations

A. Linear Equations

Examples

Solve:

1.

$$5(x-3)+2=8$$

$$5x-15+2=8$$

$$5x=21$$

$$x=4.2$$

2.

$$\frac{4(x-1)}{7} - \frac{3(1-x)}{4} = 2$$

$$16(x-1) - 21(1-x) = 56$$

$$16x - 16 - 21 + 21x = 56$$

$$37x = 93$$

$$x = 2\frac{19}{37}$$

Exercise

Solve:

1. $3(b+7) = 8(2b-3)$

2. $\frac{3}{4}c = \frac{4}{5}$

3. $(3-x) - (3x-3) = 30$

4. $\frac{1}{2}(2x+1) + \frac{1}{3}(9x-10) = 0$

5. $\frac{2(x-3)}{3} + \frac{4(1-2x)}{5} = 1$

6. $7(3x-4) - 8 = 4 - 2(x-3)$

7. $\frac{a}{2} - \frac{a}{3} = 7$

8. $\frac{2(x+5)}{3} = \frac{3(2x-3)}{4}$

B. Quadratic Equations

Examples

1. Simple factors: $x^2 + 7x + 12 = 0$
 $(x + 3)(x + 4) = 0$
 $x = -3$ or -4

2. Simple factors: $6x^2 + 7x - 3 = 0$
 $(2x + 3)(3x - 1) = 0$
 $x = -1\frac{1}{2}$ or $\frac{1}{3}$

3. Use of formula: $3x^2 - 7x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{49 - 4 \times 3 \times -2}}{6}$$

$$x = \frac{7 \pm \sqrt{73}}{6}$$

$$x = 2.59(2d.p.) \text{ or } -0.26(2d.p.)$$

Exercise

Solve:

1. $(x - 6)(x + 2) = 0$

2. $x(x + 1) = 0$

3. $x^2 + 5x + 6 = 0$

4. $x^2 - 7x = -10$

5. $x^2 = 2x$

6. $2x^2 - 3x - 2 = 0$

Solve correct to 2d.p.:

7. $x^2 + 6x + 4 = 0$

8. $2k^2 + 4k - 3 = 0$

9. $4p^2 + 7p = 6$

10. $\frac{3}{x+1} = x$

TOPIC 4 : Simultaneous Equations

By elimination method

Examples

Solve:

1.

$$\begin{array}{r} x + y = 8 \\ 4x - y = -3 \quad + \\ \hline 5x = 5 \\ x = 1 \\ y = 7 \end{array}$$

2.

$$\begin{array}{r} 5x + 3y = 2 \\ 2x + y = 0 \\ \hline 5x + 3y = 2 \\ \mathbf{x3} \quad 6x + 3y = 0 \\ \hline x = -2 \\ \hline y = 4 \end{array}$$

3.

$$\begin{array}{r} 8p - 7q = 13 \\ 3p + 2q = 28 \\ \\ \mathbf{x2} \quad 6p - 14q = 26 \\ \mathbf{x7} \quad 21p + 14q = 196 \\ \hline 37p = 222 \\ p = 6 \\ q = 5 \end{array}$$

Exercise

Solve:

1. $\begin{array}{l} 3x - 2y = 10 \\ x + 2y = 6 \end{array}$

2. $\begin{array}{l} 4x + 2y = 11 \\ 3x + 4y = 5 \end{array}$

3. $\begin{array}{l} 2x - 5y = 7 \\ 3x - 4y = 6 \end{array}$

4. $\begin{array}{l} y = 3x + 9 \\ 2x + 15 = 3y + 2 \end{array}$

5. The sum of two numbers is 32 and their difference is 6. Find the two numbers.

TOPIC 5 : Simplifying, including indices

Examples

1. $a^6 \times a^3 = a^9$

2. $m^5 \div m^3 = m^2$

3. $(x^m)^n = x^{mn}$

4. $a^3(a^4 + 2ab)$
 $= a^7 + 2a^4b$

5. $\frac{9a^2b}{c} \div \frac{27ab^2}{c^2} = \frac{9a^2b}{c} \times \frac{c^2}{27ab^2} = \frac{ac}{3b}$

Exercise

Simplify as much as possible:

A.

1. $a^4 \times a^3$

2. $a^4 \div a^3$

3. $3b^5 \div b^7$

4. $a^2(a^3 + a)$

5. $x^2(y^2 + xy + z)$

6. $(k^3)^2 + \sqrt{k^4}$

7. $\frac{16ab^2}{c^2} \times \frac{ac}{4b^2}$

8. $\frac{mn^2}{4} \div \frac{2m}{n}$

9. $\frac{a^3(a^2 - b^2)}{a^5}$

10. $\frac{3x^2(y^2 - x)}{9} \times \frac{x^4}{4}$

Simplify as much as possible:

B.

1. $a^3 \times a^{-4}$

2. $x^0 \div x^{-4}$

3. $18a^2b^2c^2 \times 2(ab)^{-1}$

4. $\frac{1}{2}c^2 \times \frac{3}{4}(pa)^2$

5. $\left(\frac{3}{4}\right)^{-2}$

6. $\left(\frac{3}{4}\right)^{-3} \div 6^{-2}$

7. $a^7 \div a^{-1}$

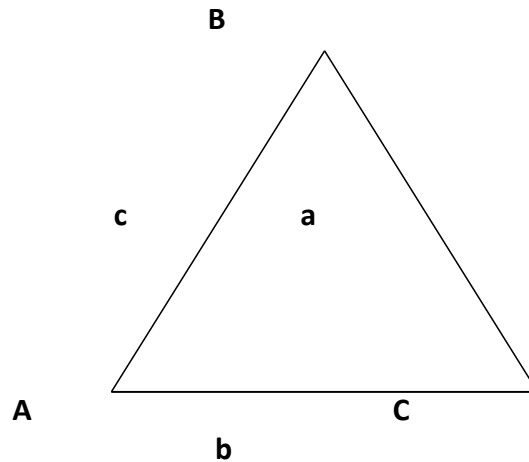
8. $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$

9. $15a^2b^7 \div 5ab^{-2}$

10. $(c^2d^4)^{\frac{1}{2}} \div c^{-1}$

TOPIC 6 : The Sine and Cosine Rules

Remember: for any triangle:



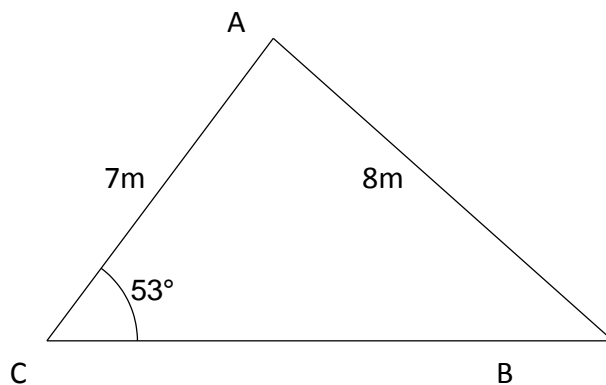
1. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \rightarrow$ The Sine Rule

2. $a^2 = b^2 + c^2 - 2bc \cos A \rightarrow$ The Cosine Rule

3. $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Examples

1. Find angle B and side BC.



$$\frac{7}{\sin B} = \frac{8}{\sin 53^\circ}$$

$$\sin B = \frac{7 \times \sin 53^\circ}{8} = 0.6988$$

$$B = 44^\circ \text{ (nearest degree)}$$

$$\text{Angle } A = 180^\circ - 44^\circ - 53^\circ = 83^\circ$$

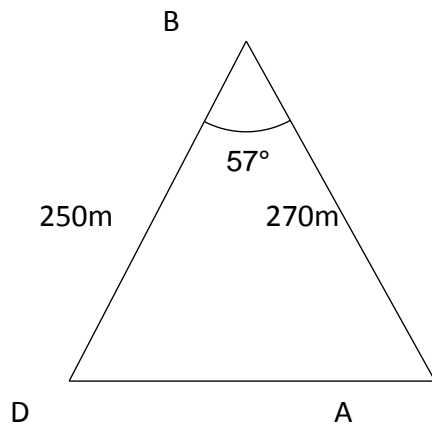
$$\therefore (BC)^2 = 7^2 + 8^2 - 2 \times 7 \times 8 \times \cos 83^\circ$$

$$BC = \sqrt{99.3506}$$

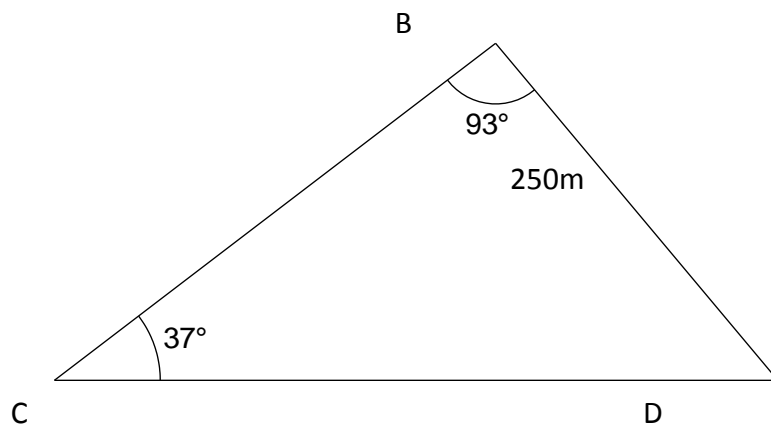
$$BC = 10.0m \text{ (3 sig. Figs.)}$$

Exercise

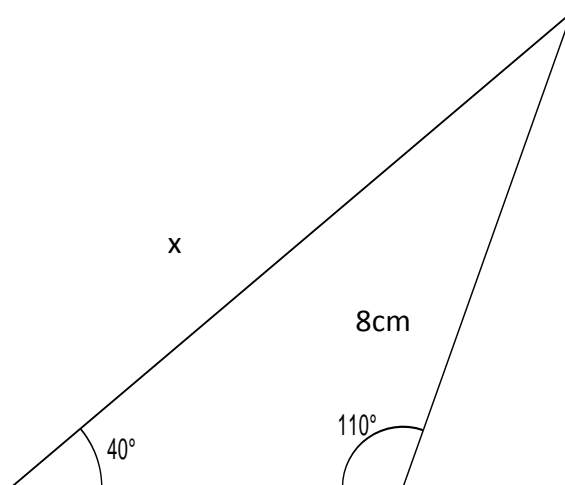
1. Calculate AD



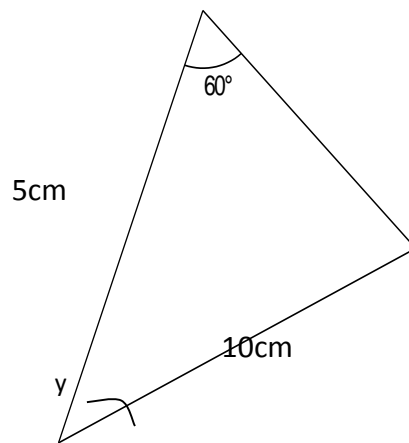
2. Calculate BC



3. Find x



4. Find y



5. In a triangle ABC, $AB = 9\text{cm}$, $BC = 5\text{cm}$ and angle $A = 24^\circ$.
- Show that there are two possible values for angle C and calculate each value;
 - Make a rough sketch to show both cases.
6. In a triangle ABC, $AB = 5\text{cm}$, $AC = 8\text{cm}$ and angle $A = 52^\circ$.
- Find BC, correct to 3 significant figures;
 - Find angle ACB, correct to the nearest degree.

We hope that you will benefit from working through this pack. Best wishes for a successful course.

Mathematics Department, Barnwell School.

SUBMISSION DATE – THURSDAY 6TH SEPTEMBER 2018